Suppression and enhancement of coherent synchrotron radiation in the presence of two parallel conducting plates

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(Received 16 April 1997)

The suppression effect of coherent synchrotron radiation due to conducting boundaries has been observed using the Tohoku University 300-MeV linac. The variation of the relative intensity of coherent synchrotron radiation with the gap between the metallic plates is well explained by the theory of Nodvick and Saxon. The emission of coherent synchrotron radiation in the long-wavelength region is suppressed more than that predicted by the theory, which indicates the possibility of other suppressing effects. [S1063-651X(98)06903-7]

PACS number(s): 29.17.+w, 29.27.Bd, 41.60.Ap, 42.72.Ai

I. INTRODUCTION

In the early days when synchrotron radiation was first observed, it was predicted that bunched electrons might radiate coherently in the region of wavelengths comparable to or longer than the bunch length. Since the coherently radiated power is proportional to the square of the number of electrons in a bunch and hence may become enormous, it was discussed that the power loss due to coherent radiation might restrict maximum energies of the high energy electron synchrotrons planned in those days. Schiff estimated coherent radiation loss and speculated that the emission of coherent synchrotron radiation might be suppressed by conducting boundaries such as a vacuum chamber [1]. Schwinger derived the general formula for radiation with conducting boundaries. Then Nodvick and Saxon solved the equation under the condition that electrons are circulating between two infinitely wide conducting plates and derived the formula for suppression of coherent synchrotron radiation [2]. In those days people seriously investigated how to suppress coherent radiation on the synchrotrons in order to obtain higher electron energies. The energy loss due to coherent radiation, however, was not observed on the constructed synchrotrons.

In 1982 Michel suggested that coherent synchrotron radiation might be observed as intense submillimeter radiation on electron storage rings [3]. In response to the speculation, some experiments to search for coherent synchrotron radiation were conducted on electron storage rings for a synchrotron radiation source. In 1984 Yarwood *et al.* reported that the intensity of synchrotron radiation in the wavelength region longer than 1 mm measured at Daresbury SRS was at least an order of magnitude higher than the intensity of a mercury lamp, which is a commonly used light source in this wavelength region [4]. They pointed out that it might be due to coherence effects. In the subsequent experiments at BESSY [5], NSLS [6], and UVSOR [7], however, no positive result was obtained for coherence effects of synchrotron radiation in the far-infrared region.

Bunch lengths of these electron storage rings are from 3 to 30 cm. On the other hand, electron bunches from linear accelerators (linacs) are 1–6 mm long and consequently it was expected that coherent synchrotron radiation was emitted in the shorter-wavelength region where detection of light was easier. In 1989 coherent synchrotron radiation was observed in the far-infrared region for the first time at the Tohoku University 300-MeV linac [8]. Coherent synchrotron radiation has been observed also on other electron linacs [9–11].

The spectrum of coherent synchrotron radiation has been measured in the wavelength region ranging from 0.16 to 3.5 mm [12,13]. The theoretical spectrum of coherent synchrotron radiation emitted by an electron bunch is given by the product of three terms; the spectrum of synchrotron radiation emitted by one electron, the square of the number of electrons in the bunch, and the bunch form factor, which reflects the shape and the size of the bunch. If the electron bunch has a Gaussian shape, the intensity of coherent synchrotron radiation is greatly enhanced at wavelengths comparable with the longitudinal bunch length and decreases as $\lambda^{-1/3}$ in the region of wavelengths longer than the peak of the spectrum, where λ is the wavelength. The measured spectrum in the longer-wavelength region, however, decreases more steeply than the theoretical prediction. It was speculated that this might be due to the suppression effect of coherent synchrotron radiation by metallic walls of the vacuum chamber.

In order to study the suppression effect, we conducted experiments using the Tohoku University linac. Two metallic plates were placed parallel to the orbit plane in a bending magnet, where the electron beam from the linac emitted co-

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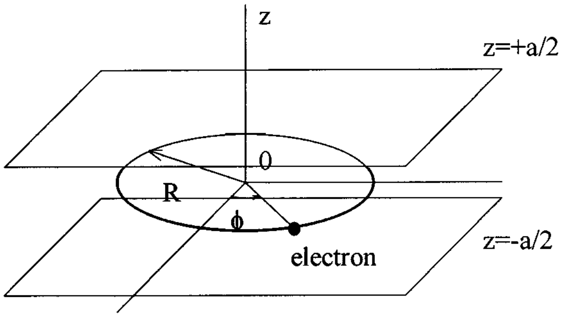

FIG. 1. Coordinate system for calculating the spectrum of coherent synchrotron radiation with the conducting boundaries.

herent synchrotron radiation. Spectra and the degree of polarization of coherent synchrotron radiation were measured as a function of the gap between the plates. Preliminary results of the experiments have been reported previously [14].

The purposes of the present study are as follows: (i) to verify experimentally if emission of coherent synchrotron radiation is suppressed by conducting boundaries, and (ii) to see if the variation of the intensity can be explained by the theory of Nodvick and Saxon for suppression of coherent synchrotron radiation.

II. THEORY

The suppression effect of coherent synchrotron radiation has been theoretically studied by Nodvick and Saxon for the electron circulating between two infinitely wide parallel conducting plates [2]. Assume that the plates are located at $z=\pm a/2$ and an electron bunch moves at the speed $v=\beta c$ along the circular orbit with the radius R on the $x-y$ plane at $z=0$, as shown in Fig. 1. The intensity of photons P_{coh} coherently radiated is given by

$$P_{\mathrm{coh}}(\beta,\lambda,a)=N(N+1)P(\beta,\lambda,a)f_b(\lambda)$$
$$\cong N^2P(\beta,\lambda,a)f_b(\lambda), \qquad (1)$$

where N is the number of electrons in the bunch, f_b the bunch form factor, and $P(\beta,\lambda,a)$ the intensity of photons radiated by a single electron. In practical units, the number of photons P radiated in the fractional band width $\Delta\lambda/\lambda$ of 1% at the wavelength λ by one electron moving in 1 mrad of the arc is given by

$$P(\beta,\lambda,a)[\text{photons}/(\text{electron}\cdot\text{mrad}\cdot 1\%\ \ \text{bandwidth})]$$
$$=9.170\times10^{-7}\frac{R[m]^2}{a[m]\beta\lambda[m]}\mathrm{Re}\left\{\sum_{j=1,3,\ldots}^{j<2\beta a/\lambda}\left[-H_n^{(1)}J_n+\frac{\beta^2}{2}(H_{n-1}^{(1)}J_{n-1}+H_{n+1}^{(1)}J_{n+1})\right]\right\}, \qquad (2)$$

where $n=2\pi R/\lambda$, J_n the Bessel function and $H_n^{(1)}$ the Hankel function of the first kind. The argument of these functions is

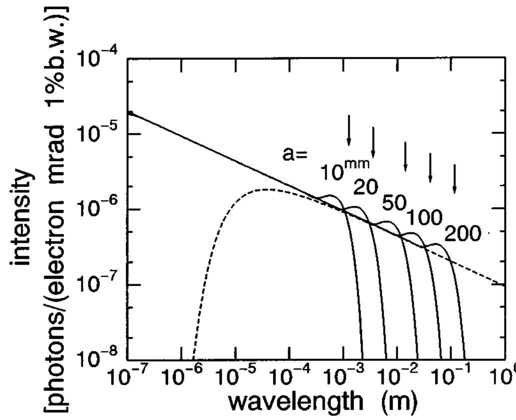

FIG. 2. Spectra of synchrotron radiation in the presence of the infinitely wide parallel conducting plates calculated with the theory of Nodvick and Saxon. The solid lines are the calculated spectra for different gaps (a) and the dashed line is the spectrum of synchrotron radiation emitted by a 50-MeV electron in a bending magnet with a radius of 2.44 m. The arrows indicate the critical wavelengths given by Eq. (5).

$$\gamma_{nj}R=\left[(n\beta)^2-\left(\frac{j\pi R}{a}\right)^2\right]^{-1/2}=\frac{\pi R}{a}\left[\left(\frac{2a\beta}{\lambda}\right)^2-j^2\right]^{-1/2}. \qquad (3)$$

If the speed of the electron approximates the speed of light, $P(\beta=1,\lambda,a)$, which is hereafter denoted by $P(\lambda,a)$, is given by

$$P(\lambda,a)=1.946\times10^{-7}\frac{R^2}{a\lambda}\left\{\sum_{j=1,3,\ldots}^{j<2a/\lambda}(\xi_j/n)^4[K_{1/3}^2(\xi_j^3/3n^2)+K_{2/3}^2(\xi_j^3/3n^2)]\right\}, \qquad (4)$$

where $\xi_j=j\pi R/a$, and $K_{1/3}$ and $K_{2/3}$ are the modified Bessel functions. The solid lines in Fig. 2 show spectra of $P(\lambda,a)$ calculated with Eq. (4). The theory predicts that the emission of synchrotron radiation is suppressed in the long-wavelength region. As the wavelength becomes shorter than the gap between the conducting boundaries, the intensity first increases more than that of ordinary synchrotron radiation without boundaries [15], which is shown by the dotted line in Fig. 2 for an electron of 50 MeV energy, and then approaches it again. The wavelength around with the suppression effect appears is given by [16]

$$\lambda_c=2a\sqrt{a/R}, \qquad (5)$$

where a is the gap and R is the bending radius. Here we call it the critical wavelength. It is indicated in Fig. 2 by arrows for several gaps. The physical meaning of the critical wavelength may be intuitively understood as follows. The angular opening of the emitted synchrotron radiation is given by [1]

$$\delta\theta\sim n^{-1/3}=\left(\frac{\lambda}{2\pi R}\right)^{1/3}. \qquad (6)$$

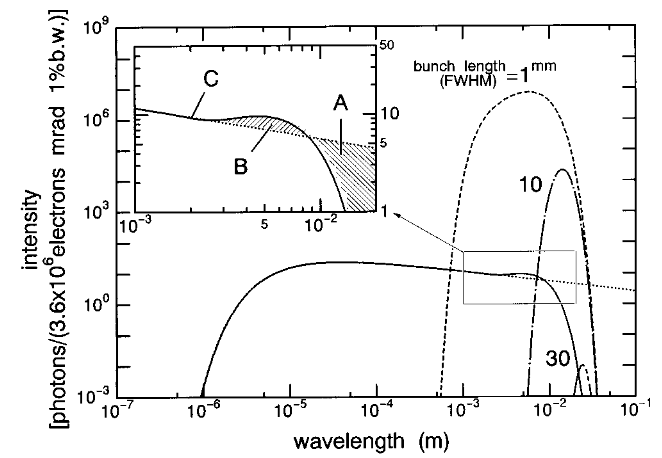

FIG. 3. Calculated spectra of incoherent and coherent synchrotron radiation in the presence of the infinitely wide conducting plates emitted by an electron bunch with different lengths. The solid line is the spectrum of incoherent synchrotron radiation and the other lines are spectra of coherent synchrotron radiation. The electron energy is 50 MeV and the number of electron in the bunch is 3.6×10^{6}. The gap between the plates is 40 mm and the radius of the bending magnet is 2.44 m. The total energy radiated as coherent synchrotron radiation strongly depends on the bunch length. The inset shows the portion of the spectrum of incoherent synchrotron radiation. The numerical calculation shows that the sum of the power radiated in the areas A and C is equal to that in the area B, which means that the total energy radiated by an electron with the metallic plates is the same as that without them.

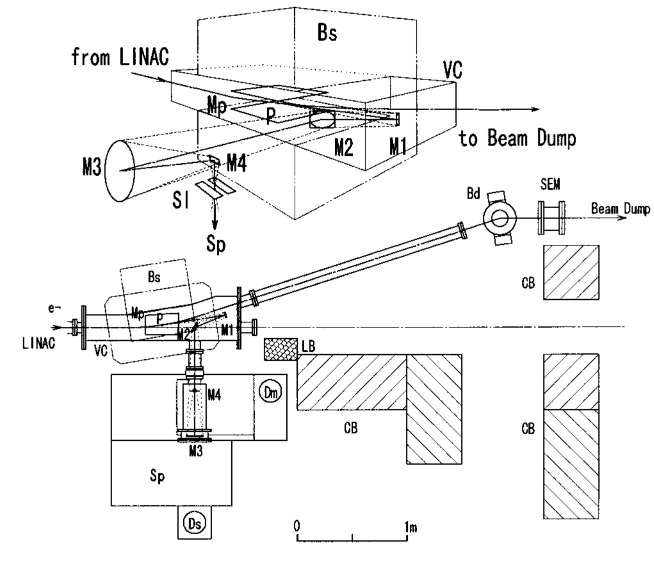

FIG. 4. Schematic layout of the experimental setup. Bs and Bd, bending magnets; Mp, metallic plates; P, light emitting point; $M1$, $M2$, and $M4$, plane mirrors; $M3$, spherical mirror; Sl, slit; Sp, spectrometer; Ds and Dm, infrared detectors; SEM, secondary emission monitor (current monitor); VC, vacuum chamber; CB, concrete block; LB, lead block.

From a simple argument based on the Fourier transformation, the effective transverse size of the light source d may be given by [17]

$$d \gtrsim \lambda/\delta\theta \sim \lambda^{2/3}R^{1/3}. \tag{7}$$

Solving Eq. (7) with respect to λ, one obtains

$$\lambda \lesssim d\sqrt{d/R}. \tag{8}$$

Aside from the numerical factor, the critical wavelength defined by Eq. (5) is the wavelength at which the effective source size d is equal to the gap a.

As the wavelength becomes shorter, the intensity calculated with Eq. (4) increases monotonously aside from the spectrum of synchrotron radiation without the boundaries, because we have assumed in Eq. (4) that the speed of the electron is equal to that of light. The proper spectrum of synchrotron radiation with the boundaries is obtained by connecting spectra with and without the boundaries at an intermediate wavelength. The solid line in Fig. 3 shows such a spectrum of incoherent synchrotron radiation for $R = 2.44$ m and $a = 40$ mm, which is the typical vertical size of the vacuum chamber in a bending magnet. Coherent synchrotron radiation is suppressed in the long-wavelength region because incoherent synchrotron radiation is suppressed by the metallic plates. The inset in Fig. 3 is a portion of the spectrum near the wavelength given by Eq. (5). The intensity of synchrotron radiation in regions A and C are suppressed by the metallic plates, whereas the intensity in region B is enhanced compared with synchrotron radiation without the plates. As the result of numerical integration, it turns out that the sum of the power radiated in areas A and C is equal to that in area B, which indicates that the total energy radiated by a single electron does not change due to the presence of the conducting boundaries. The intensity of coherent synchrotron radiation, however, depends on the bunch form factor as given in Eq. (1). The calculated spectra of coherent synchrotron radiation emitted with the same boundary conditions by electron bunches which have the Gaussian shape with bunch lengths of 1, 10, and 30 mm (the full width at half maximum, FWHM) are also shown in Fig. 3. When the bunch length is longer than the wavelength given by Eq. (5), the total power of coherent synchrotron radiation emitted by the bunch drastically decreases. Electron bunches accelerated by a linac are typically 1–10 mm long. While electron bunches of storage rings are longer than 30 mm and the emission of coherent synchrotron radiation is strongly suppressed for the same gap. The total energy loss by an electron bunch due to emission of coherent synchrotron radiation drastically changes with the gap between the conducting boundaries and the bunch length.

III. EXPERIMENTAL METHOD

The experiment to study the effects of conducting boundaries on the emission of coherent synchrotron radiation has been carried out at the Tohoku University 300-MeV linac. The experimental setup is shown schematically in Fig. 4. The electron beam accelerated to 50 MeV by the linac was injected to the bending magnet Bs. A bending radius of the magnet was 2.44 m and a magnetic field was 68.6 mT. The energy spread of the electron beam was 0.2%. The time duration of the beam pulse was 2 μs long and its repetition rate was 300 pulses/s. One pulse is made up of approximately 5700 electron bunches and one bunch contains

3.6×10^6 electrons. The longitudinal bunch length was approximately 1.7 mm, which corresponds to the phase angle of 5.5 degrees in the accelerating rf of 2.856 GHz [18].

At the light emitting point P two aluminum parallel plates Mp 1 mm thick were installed in the bending magnet Bs such that the electron beam went through the center of the gap between the plates. The gap could be varied from 81 down to 12 mm. The shape of the plates was trapezoidal. They are 300 mm long along the beam axis, 180 mm wide at the upstream side, and 200 mm wide at the downstream side. The plates were bent outward at 50 mm from the down stream side by an angle of 100 mrad so that they did not restrict the acceptance angle of the detection system of light. The vertical cross section of the aluminum vacuum chamber at the light emitting point P is 270 mm wide and 180 mm high.

Synchrotron radiation emitted between the metallic plates was focused and led to the monochromator Sp with the spherical mirror $M3$ shown in Fig. 4, which extends an acceptance angle of 70 mrad to the light emitting point. We used two types of monochromators. The light intensity at a fixed wavelength was measured as a function of the gap with a grating-type far-infrared spectrometer [12]. The degree of polarization of coherent synchrotron radiation was measured with this spectrometer equipped with an additional polarizer in it. The spectrum of synchrotron radiation was measured at a fixed gap with a polarizing interferometer [13]. The light came out of the monochromator was detected with a liquid-helium-cooled silicon bolometer Ds. Another detector denoted by Dm was used to monitor the intensity of the incident light. In order to cut the stray light reflected by the metallic plates, the slit Sl was placed at the focal point of the mirror $M3$. When the slit was used, its width was set at 6 mm, which was determined such that the spectrum of light from a mercury lamp placed at the light emitting point P did not change when the gap was varied from 81 to 24 mm.

The overall efficiency of the detection system was calibrated with a black body source. Since the intensity of coherent synchrotron radiation is not proportional to the beam current, the measured data has to be corrected for the fluctuation of the beam current. The intensity of coherent synchrotron radiation was hence measured at wavelengths of 1.6, 3.0, and 4.0 mm as a function of the average beam current over the range from 0.5 to 5 μA. The measured beam current dependence of the intensity was well fitted with the function proportional to the square of the beam current, as expected for coherent synchrotron radiation. The root-mean-square deviation of the measured data from the fitted function was taken as the error of the measured data, which includes the temporal stability and the beam current dependence of the bunch shape. This error is shown in the figures below.

IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

Figure 5 shows spectra measured with the polarizing interferometer for gaps between the metallic plates from 81 down to 15 mm. The solid lines show spectra measured with the slit for cutting stray light and the dash-dotted lines show those without it. When the slit was not used, the measured spectra might include stray light due to reflection by the

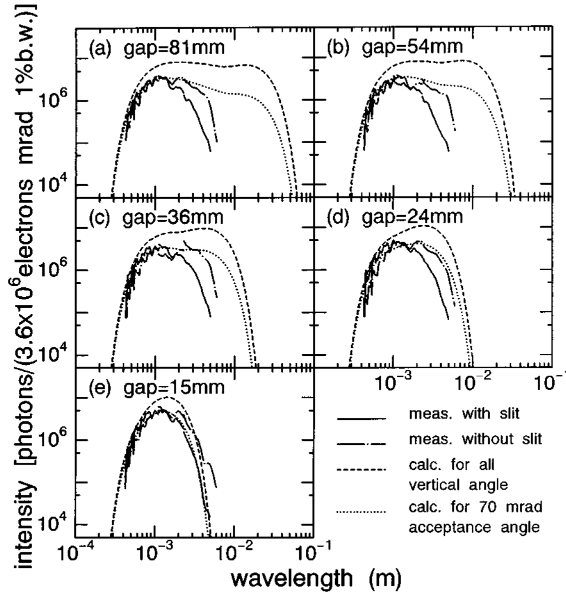

FIG. 5. Spectra of coherent synchrotron radiation for gaps between the metallic plates from 81 down to 15 mm. The solid lines are the measured spectra with the slit and the dash-dotted lines are those without it. The dashed lines are the spectra calculated with the theory of Nodvick and Saxon and the dotted lines are those with an acceptance angle of 70 mrad. In this calculation, the bunch shape is assumed to be the Gaussian shape with the bunch length (FWHM) of 0.3 mm.

metallic plates. When it was used, on the other hand, a part of coherent synchrotron radiation might be lost due to diffraction by the slit especially in the long-wavelength region. Since it is very difficult to exclude the diffraction effect in the long-wavelength region, both of the spectra with and without the slit are shown in Fig. 5. The dashed lines are theoretical spectra of coherent synchrotron radiation including the suppression effect calculated with Eqs. (1) and (4). In the calculation, we assumed that the bunch had the Gaussian shape with the bunch length (full width at half-maximum) of 0.3 mm, which gives the best fit to the short-wavelength side of the experimental spectra where the intensity of coherent synchrotron radiation rises rapidly. The assumed bunch length is shorter than the previously estimated bunch length of 1.7 mm. The measured spectra were obtained from the number of photons emitted in the circular cone with the full width of 70 mrad, while the theoretical spectra denoted by the dashed lines show the total number of photons integrated over the vertical angle. In this wavelength region, the vertical angular distribution of incoherent synchrotron radiation has the width comparable to or larger than the vertical acceptance angle of detection system, 70 mrad. If we assume that the vertical angular distribution of coherent synchrotron radiation is the same as that of incoherent synchrotron radiation, we obtain the theoretical spectra in the same condition of the acceptance angle denoted by the dotted lines.

Figure 6 shows the degree of polarization of coherent synchrotron radiation as a function of the gap at wavelengths from 1.6 to 5 mm, which was measured with the grating-type far-infrared spectrometer and the slit for cutting stray light. The degree of polarization is independent of the gap and nearly constant except for that at $\lambda=5$ mm. We think that the

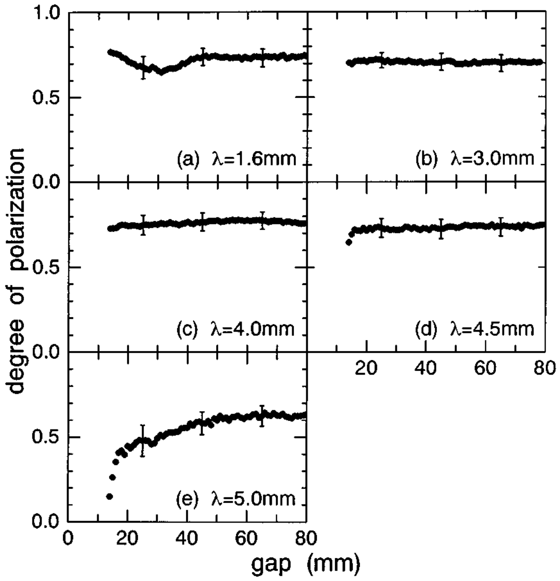

FIG. 6. Degree of polarization of coherent synchrotron radiation as a function of the gap between the metallic plates for wavelengths from 1.6 to 5.0 mm. The degree of polarization is defined as $P=(I_h-I_v)/(I_h+I_v)$ where I_h and I_v are the intensity with electric vectors parallel and perpendicular to the orbital plane, respectively.

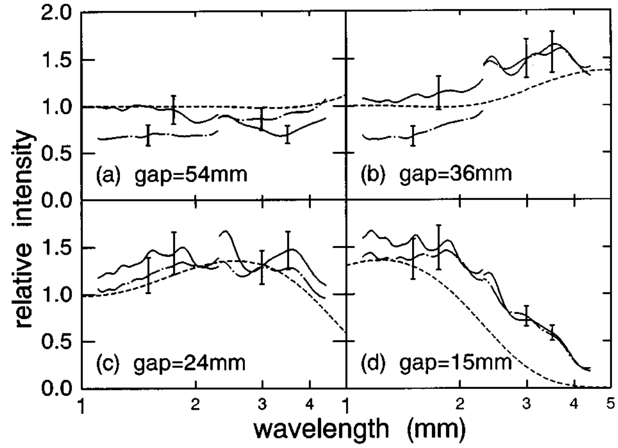

FIG. 7. Spectrum ratios of coherent synchrotron radiation for gaps from 54 to 15 mm relative to the reference spectrum at gap =81 mm. The solid lines show measured spectra with the slit and the dash-dotted lines show those without it. The dashed lines show spectra calculated with the theory of Nodvick and Saxon.

degree of polarization at λ=5 mm decreased due to mixing of stray light and that it was not intrinsic. The degree of polarization of coherent synchrotron radiation measured at wavelengths from 1.6 to 4.5 mm is not equal to that previously measured in the shorter wavelength region from 0.333 to 1.5 mm [19]. In the previous measurement a different far-infrared spectrometer (Hitachi FIS-3) was used, and the metallic plates in the vacuum chamber and the slit for cutting stray light did not exist.

The experimental spectra in Fig. 5 fall at wavelengths shorter than those predicted by the theory of Nodvick and Saxon. It seems that, contrary to the prediction of the theory, the critical wavelength does not vary greatly with the gap. There may be another or other factors that distort the spectrum more than the suppression effect by the conducting boundaries. They may depend on the wavelength, but should not depend on the gap.

We therefore include another factor denoted by $g(\lambda)$ in the right-hand side of Eq. (1) as

$$P'_{\mathrm{coh}}(\beta,\lambda,a)=N^2P(\lambda,a)f_b(\lambda)g(\lambda). \tag{9}$$

Contrary to the bunch form factor $f_b(\lambda)$, $g(\lambda)$ should be equal to one in the shorter-wavelength region and should decrease, independently of the gap, down to zero in the longer-wavelength region in order to explain the falls of the measured spectra shown in Fig. 5. Such a factor may originate from another suppression effect or measurement conditions such as the slit of the spectrometer, the slit *Sl* in Fig. 4 for cutting stray light, and a finite solid angle of detection subtend by the detector. Although these measurement conditions give the same tendency, it seems that they are not enough to explain the discrepancies between the measured spectra and the calculated ones as shown in Fig. 5.

In order to exclude $g(\lambda)$, let us take the ratio of the spectrum to that at the reference gap a_0 as

$$\frac{P'_{\mathrm{coh}}(\beta,\lambda,a)}{P'_{\mathrm{coh}}(\beta,\lambda,a_0)}=\frac{P(\lambda,a)}{P(\lambda,a_0)}. \tag{10}$$

Equation (10) does not depend on any of the bunch shape, size and the other factors that are independent of the gap, so that the experimental spectrum ratio on the left-hand side of Eq. (10) can be directly compared with theoretical one on the right-hand side calculated with the theory of Nodvick and Saxon. The emission of coherent synchrotron radiation is enhanced compared with the emission at the reference gap a_0, when the spectrum ratio is larger than one, and suppressed when it is less than one.

Figure 7 shows the spectrum ratios measured with the polarizing interferometer for gaps from 54 to 15 mm together with the calculated spectrum ratios. The reference gap is chosen to be $a_0=81$ mm where the influence of the metallic plates may be neglected. The solid lines are those with the slit, the dash-dotted lines are those without it, and the dash lines are calculated spectrum ratios. The calculated spectrum ratio at the gap of 54 mm shows no suppression and enhancement at the wavelengths shorter than 4 mm. As the gap decreases, however, the emission is first enhanced and then suppressed. The measured spectrum ratios with and without the slit have similar trends to the calculated ones within the experimental error bars. The variation of the spectrum ratio with the gap is explained by the suppression theory derived by Nodvick and Saxon.

Figure 8 shows relative intensities of coherent synchrotron radiation as a function of the gap measured at wavelengths from 1.6 to 5 mm with the grating-type far-infrared spectrometer as well as calculated ones. The reference gap is $a_0=80$ mm. The solid circles denote intensities measured with the slit and open circles without it. The dashed lines are calculated values. If there is no suppression effect due to the metallic plates, the relative intensity would be independent

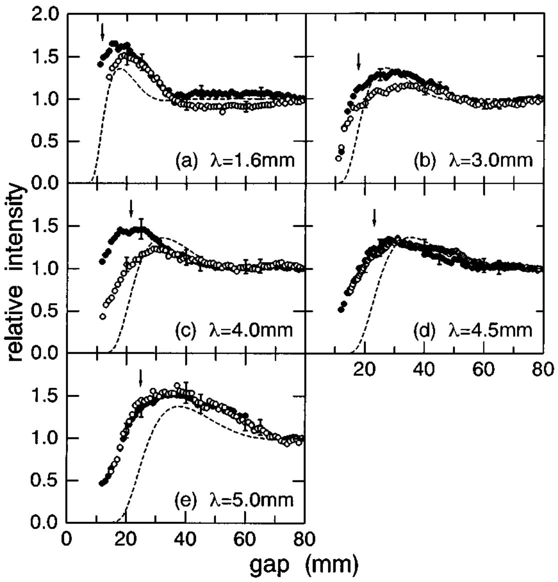

FIG. 8. Relative intensities of coherent synchrotron radiation as a function of the gap between the metallic plates for wavelengths from 1.6 to 5.0 mm. The intensities are normalized with the intensity at the gap of 80 mm for each wavelength. The solid circles denote intensities measured with the slit for cutting stray light and the open circles without it. The dashed lines are calculated values. The arrows indicate the critical wavelengths given by Eq. (5).

of the gap and equal to one. As the gap becomes small, however, the measured relative intensities shown in Fig. 8 first increase more than one and then rapidly decrease less than one. The experimental results clearly show that the emission of coherent synchrotron radiation is not only suppressed but also enhanced by the metallic plates. The agreement between the experimental results and the relative intensities calculated by the theory of Nodvick and Saxon is fairly good. The poor agreement in the smaller gap region may be ascribed to finite dimensions of the metallic plates, to another boundary conditions of the side walls in the vacuum duct, or to coherent diffraction radiation emitted by the electron beam at the edges of the metallic plates [20].

V. SUMMARY AND CONCLUSIONS

The emission of coherent synchrotron radiation was studied in the presence of conducting boundaries. The suppression effect of coherent radiation due to the metallic plates was observed. They not only suppressed the emission of coherent synchrotron radiation but also enhanced it. The intensity became 1.3–1.5 times higher in a certain region of the gap between the plates. The degree of polarization of coherent synchrotron radiation was constant and did not depend on the gap for wavelengths between 1.6–4.5 mm. The spectrum of coherent synchrotron radiation with the conducting boundaries was calculated with the theory of Nodvick and Saxon. It was found by the numerical calculation that the total energy of synchrotron radiation emitted by a single electron did not change due to the presence of the conducting boundaries. The variations of the relative intensity of coherent synchrotron radiation with the wavelength and with the gap were explained by the theory of Nodvick and Saxon. The theory, on the other hand, could not account for the intensity reduction in the region of wavelengths longer than 2 mm, which indicates the possibility of another or other suppressing effects.

The results of the present experiment are applicable to the beam tube design for accelerators with very short bunches in order to suppress the energy loss due to emission of coherent synchrotron radiation, and also to that for bending magnets of storage rings in order to enhance the light intensity at a certain wavelength.

ACKNOWLEDGMENTS

This work was supported in part by a Grant-in-Aid for General Scientific Research, Japan, for Developmental Scientific Research and for Encouragement of Young Scientists of the Ministry of Education, Science, Sports and Culture. Two authors (R.K. and T.N.) express their sincere thanks to Professor G. Isoyama of Osaka University for critically reviewing the manuscript, and are grateful to Dr. R. L. Warnock for his valuable suggestions about the theory. The authors thank Mr. T. Tsutaya for his technical support for our making optical elements and Dr. O. Konno for his help with our designing and making the vacuum chamber.

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